## One Dimensional Arrays ( array )

## Definition

An instance $A$ of the parameterized data type array<E> is a mapping from an interval $I=[a . . b]$ of integers, the index set of $A$, to the set of variables of data type $E$, the element type of $A$. $A(i)$ is called the element at position $i$.

```
#include < LEDA/array.h >
```


## Types

array<E>::item the item type.
array<E>::value_type the value type.

## Creation

$$
\begin{array}{ll}
\text { array<E> } A(\text { int } a, \text { int b) } & \text { creates an instance } A \text { of type array<E> with index set }[a . . b] . \\
\text { array<E> } A(\text { int n) } & \text { creates an instance } A \text { of type array<E> with index set }[0 . . n-1] . \\
\text { array<E> A } & \text { creates an instance } A \text { of type array<E> with empty index set. }
\end{array}
$$

## Special Constructors

array<E> A(int low, Ex, E creates an instance $A$ of type array<E> with index set [low, low y) $+1]$ initialized to $[x, y]$.
array<E> A(int low, Ex, E creates an instance $A$ of type array<E> with index set [low, low $y, E$ w) $\quad+2]$ initialized to $[x, y, w]$.
array<E> A(int low, E x, E y, E z, E w)
creates an instance $A$ of type array<E> with index set [low, low $+3]$ initialized to $[x, y, z, w]$.

## Operations

## Basic Operations

E\& A[int x] returns $A(x)$.
Precondition $a<=x<=b$.
int A.low() returns the minimal index $a$ of $A$.
int A.high() returns the maximal index $b$ of $A$.
int A.size() returns the size $(b-a+1)$ of $A$.

Iteration STL compatible iterators are provided when compiled with -
DLEDA_STL_ITERATORS (see LEDAROOT/demo/st//array.c for an example).

## Implementation

Arrays are implemented by C++vectors. The access operation takes time $O(1)$, the sorting is realized by quicksort (time $O(n \log n)$ ) and the binary_search operation takes time $O(\log n)$, where $n=b-a+1$. The space requirement is $O\left({ }^{*}\right.$ sizeof $\left.(E)\right)$.

## Linear Lists ( list )

## Definition

An instance $L$ of the parameterized data type list<E> is a sequence of items (list item). Each item in $L$ contains an element of data type $E$, called the element type of $L$. The number of items in $L$ is called the length of $L$. If $L$ has length zero it is called the empty list. In the sequel $<x\rangle$ is used to denote a list item containing the element $x$ and $L[]$ is used to denote the contents of list item $i$ in $L$.
\#include < LEDA/list.h >

## Types

list<E>::item the item type.
list<E>::value_type the value type.

## Creation

list<E> L creates an instance $L$ of type list<E> and initializes it to the empty list.

## Operations

## Access Operations

| int | L.length() | returns the length of $L$. |
| :---: | :---: | :---: |
| int | L.size() | returns L.length(). |
| bool | L.empty() | returns true if $L$ is empty, false otherwise. |
| list_item | L.first() | returns the first item of $L$ (nil if $L$ is empty). |
| list_item | L.last() | returns the last item of $L$. (nil if $L$ is empty) |
| list_item | L.succ(list_item it) | returns the successor item of item $i t$, nil if $i t=L$.last(). Precondition it is an item in $L$. |
| list_item | L.pred(list_item it) | returns the predecessor item of item it, nil if it = L.first(). Precondition it is an item in $L$. |
| E | L.inf(list_item it) | returns L.contents(it). |
| E | L.front() | returns the first element of $L$, i.e. the contents of $L$.first(). Precondition $L$ is not empty. |
| E | L.head() | same as L.front(). |
| E | L.back() | returns the last element of $L$, i.e. the contents of $L$.last(). Precondition $L$ is not empty. |
| E | L.tail() | same as L.back(). |

## Update Operations

| list_item L.push(E x) | adds a new item $<x>$ at the front of $L$ and returns it <br> $(L . i n s e r t(x, L$. first ()$, L E D A::$ before $))$. |
| :--- | :--- |
| list_item L.push_front(E x) | same as $L . p u s h(x)$. |
| list_item L.append(E x) | appends a new item $<x>$ to $L$ and returns it |


|  |  | (L.insert( $(x, L . l a s t(), L E D A::$ after ) ). |
| :---: | :---: | :---: |
| list_item | L.push_back(Ex) | same as L.append( $x$ ). |
| E | L.pop() | deletes the first item from $L$ and returns its contents. Precondition $L$ is not empty. |
| E | L.pop_front() | same as L.pop(). |
| E | L.Pop() | deletes the last item from $L$ and returns its contents. Precondition $L$ is not empty. |
| E | L.pop_back() | same as L.Pop(). |
| E | L.del_item(list_item it) | deletes the item it from $L$ and returns its contents $L[i f]$. Precondition it is an item in $L$. |
| E | L.del(list_item it) | same as L.del_item(it). |
| void | L.erase(list_item it) | deletes the item it from $L$. Precondition it is an item in $L$. |
| void | L.clear() | makes $L$ the empty list. |

## Operators

| E\& | L[list_item it] | returns a reference to the contents of it. |
| :---: | :---: | :---: |
| list_item | $L+=E x$ | same as L.append(x); returns the new item. |
| ostream\& | ostream\& out << L | same as L.print(out); returns out. |
| istream\& | istream\& in >> lis | same as L.read(in)); returns in. |

## Iteration

forall_items $(i t, L)$ \{ "the items of $L$ are successively assigned to it" \}
forall $(x, L)$ \{ "the elements of $L$ are successively assigned to $x "\}$
STL compatible iterators are provided when compiled with -DLEDA_STL_ITERATORS (see LEDAROOT/demo/st//list.c for an example).

## Implementation

The data type list is realized by doubly linked linear lists. All operations take constant time except for the following operations: search and rank take linear time $O(n)$, item $(i)$ takes time $O(i)$, bucket_sort takes time $O(n+j-i)$ and sort takes time $O\left(n^{*} c^{*} \log n\right)$ where $c$ is the time complexity of the compare function. $n$ is always the current length of the list.

## Graphs ( graph )

## Definition

An instance $G$ of the data type graph consists of a list $V$ of nodes and a list $E$ of edges (node and edge are item types). Distinct graphs have disjoint node and edge lists. The value of a variable of type node is either the node of some graph, or the special value nil (which is distinct from all nodes), or is undefined (before the first assignment to the variable). A corresponding statement is true for the variables of type edge.
\#include < LEDA/graph.h >

## Creation

graph $G$ creates an object $G$ of type graph and initializes it to the empty directed graph.

## Operations

## a) Access operations

| int | G.outdeg(node v) | returns the number of edges adjacent to node $v$ (\| adj_edges( $v) \mid$ ). |
| :---: | :---: | :---: |
| int | G.indeg(node v) | returns the number of edges ending at $v(\mid$ in_edges $(v) \mid)$ if $G$ is directed and zero if $G$ is undirected). |
| int | G.degree(node v) | returns outdeg $(v)+$ indeg $(v)$. |
| node | G.source(edge e) | returns the source node of edge e. |
| node | G.target(edge e) | returns the target node of edge e. |
| node | G.opposite(node v, edg |  |
|  |  | returns target(e) if $v=$ source(e) and source(e) otherwise. |
| int | G.number_of_nodes() | returns the number of nodes in $G$. |
| int | G.number_of_edges() | returns the number of edges in $G$. |
| list<node> | G.all_nodes() | returns the list $V$ of all nodes of $G$. |
| list<edge> | G.all_edges() | returns the list $E$ of all edges of $G$. |
| node | G.first_node() | returns the first node in $V$. |
| node | G.last_node() | returns the last node in $V$. |
| node | G.choose_node() | returns a random node of $G$ (nil if $G$ is empty). |
| edge | G.first_edge() | returns the first edge in $E$. |
| edge | G.last_edge() | returns the last edge in $E$. |
| edge | G.choose_edge() | returns a random edge of $G$ (nil if $G$ is empty). |
| bool | G.is_directed() | returns true iff $G$ is directed. |
| bool | G.is_undirected() | returns true iff $G$ is undirected. |
| bool | G.empty() | returns true iff $G$ is empty. |

## b) Update operations

| node | G.new_node() adds a new node to $G$ and returns it. |
| :--- | :--- |
| edge | G.new_edge(node $v$, node $w)$ |

adds a new edge $(v, w)$ to $G$ by appending it to $a d j$ edges( $v$ ) and to in_edges( $w$ ) (if $G$ is directed) or adj_ edges( $w$ ) (if $G$ is undirected), and returns it.
void G.hide_edge(edge e)
bool G.is_hidden(edge e)
list<edge> G.hidden_edges()
void G.restore_edge(edge
e)
void G.restore_all_edges()
void G.hide_node(node v) removes edge $e$ temporarily from $G$ until restored by G.restore_edge(e).
returns true if $e$ is hidden and false otherwise.
returns the list of all hidden edges of $G$.
restores e by appending it to adj_edges_(source(e)) and to in_edges(target(e)) ( adj_edges(target(e)) if $G$ is undirected). Precondition $e$ is hidden and neither source(e) nor target(e) is hidden.
restores all hidden edges.
removes node $v$ temporarily from $G$ until restored by G.restore_node( $v$ ). All non-hidden edges in adj_edges( $v$ ) and in_edges( $v$ ) are hidden too.
void G.hide_node(node $v$, list<edge>\& h_edges)
as above, in addition, the list of leaving or entering edges which are hidden as a result of hiding $v$ are appended to h_edges.
bool G.is_hidden(node v) returns true if $v$ is hidden and false otherwise.
list<node> G.hidden_nodes() returns the list of all hidden nodes of $G$.
void G.restore_node(node restores $v$ by appending it to the list of all nodes. Note that
v) no edge adjacent to $v$ that was hidden by G.hide_node( $v$ ) is restored by this operation.
void G.restore_all_nodes() restores all hidden nodes.
void G.del_node(node v) deletes $v$ and all edges incident to $v$ from $G$.
void G.del_edge(edge e) deletes the edge e from $G$.
void G.del_all_nodes() deletes all nodes from $G$.
void G.del_all_edges() deletes all edges from $G$.
void G.sort_nodes(node array<T>A)
the nodes of $G$ are sorted according to the entries of node_array $A$ (cf. section Node Arrays).
Precondition $T$ must be numerical.
void G.sort_edges(edge array<T>A)
the edges of $G$ are sorted according to the entries of edge_array $A$ (cf. section Edge Arrays). Precondition $T$ must be numerical.
void G.make_undirected() makes $G$ undirected by appending in_edges(v) to adj _ edges( $v$ ) for all nodes $v$.
void $\quad$ G.make_directed() makes $G$ directed by splitting adj_edges(v) into out _ edges( $v$ ) and in_edges( $v$ ).
void $\quad$ G.clear() makes $G$ the empty graph.

## f) I/O Operations

void G.print_node(node v, ostream\& $\mathrm{O}=$ cout)
prints node $v$ on the output stream $O$.
void G.print_edge(edge e, ostream\& $\mathrm{O}=$ cout)
prints edge $e$ on the output stream $O$. If $G$ is directed $e$ is represented by an arrow
pointing from source to target. If $G$ is undirected $e$ is printed as an undirected line segment.
void G.print(string s, ostream\& $O=$ cout $)$
prints $G$ with header line $s$ on the output stream $O$.
void G.print(ostream\& $\mathrm{O}=$ cout)
prints $G$ on the output stream $O$.
g) Non-Member Functions

| node | source(edge e) | returns the source node of edge $e$. |
| :--- | :--- | :--- |
| node | target(edge e) | returns the target node of edge $e$. |
| graph* | graph_of(node v) | returns a pointer to the graph that $v$ belongs to. |

h) Iteration

All iteration macros listed in this section traverse the corresponding node and edge lists of the graph, i.e. they visit nodes and edges in the order in which they are stored in these lists.
forall_nodes $(v, G)$
\{"the nodes of $G$ are successively assigned to $v^{\prime \prime}$ \}
forall_edges $(e, G)$
\{ "the edges of $G$ are successively assigned to $e$ " \}
forall_rev_nodes( $v, G$ )
\{ "the nodes of $G$ are successively assigned to $v$ in reverse order" \}
forall_rev_edges(e, $G$ )
\{"the edges of $G$ are successively assigned to $e$ in reverse order" \}
forall_adj_edges(e, w)
\{ "the edges adjacent to node $w$ are successively assigned to e" \}
forall_out_edges $(e, w)$ a faster version of forall_adj_edges for directed graphs.
forall_in_edges $(e, w)$
\{ "the edges of in_edges(w) are successively assigned to $e^{\prime \prime}$ \}
forall_inout_edges( $(e, w)$
\{ "the edges of adj_edges $(w)$ and in_edges $(w)$ are successively assigned to $e^{"}$ \}
forall_adj_nodes( $v, w)$
\{ "the nodes adjacent to node $w$ are successively assigned to v" \}

## Implementation

Graphs are implemented by doubly linked lists of nodes and edges. Most operations take constant time, except for all_nodes, all_edges, del_all_nodes, del_all_edges, make_map, make_planar_map, compute_faces, all_faces, make_map, clear, write, and read which take time $O(n+m)$, and adj_edges, adj_nodes, out_edges, in_edges, and adj_faces which take time O(output size) where $n$ is the current number of nodes and $m$ is the current number of edges. The space requirement is $O(n+m)$.

## Parameterized Graphs (GRAPH)

## Definition

A parameterized graph $G$ is a graph whose nodes and edges contain additional (user defined) data. Every node contains an element of a data type vtype, called the node type of $G$ and every edge contains an element of a data type etype called the edge type of $G$. We use $<v$, $w, y\rangle$ to denote an edge $(v, w)$ with information $y$ and $\langle x\rangle$ to denote a node with information $x$.
\#include < LEDA/graph.h >

## Creation

GRAPH<vtype,etype> G creates an instance $G$ of type GRAPH<vtype,etype> and initializes it to the empty graph.

## Operations

| vtype | G.inf(node v) | returns the information of node $v$. |
| :---: | :---: | :---: |
| const vtype\& | G[node v] | returns a reference to $\operatorname{G} \cdot \inf (v)$. |
| etype | G.inf(edge e) | returns the information of edge e. |
| const etype\& | G[edge e] | returns a reference to $\operatorname{G} . \inf (e)$. |
| node | G.new_node(vtype x) | adds a new node $<x>$ to $G$ and returns it. |
| edge | G.new_edge(node | node w, etype x) |

adds a new edge $\langle v, w, x\rangle$ to $G$ by appending it to adj _ edges( $v$ ) and to in _edges( $w$ ) and returns it.
void G.sort_nodes(list<node> vl)
makes $v /$ the node list of $G$.
Precondition $v /$ contains exactly the nodes of $G$.
void G.sort_edges(list<edge> el)
makes el the edge list of $G$.
Precondition el contains exactly the edges of $G$.
void G.sort_nodes() the nodes of $G$ are sorted increasingly according to their contents.
Precondition vtype is linearly ordered.
void
G.sort_edges() the edges of $G$ are sorted increasingly according to their contents.
Precondition etype is linearly ordered.

## Implementation

Parameterized graphs are derived from directed graphs. All additional operations for manipulating the node and edge entries take constant time.

## Undirected Graphs ( ugraph )

## Definition

An instance $U$ of the data type ugraph is an undirected graph as defined in section Graphs.

```
#include < LEDA/ugraph.h >
```


## Creation

| ugraph $U$ | creates an instance $U$ of type ugraph and initializes it to the empty <br> undirected graph. |
| :---: | :--- |
| ugraph$U($ graph  <br> G) creates an instance $U$ of type ugraph and initializes it with an undirected <br> copy of $G$.  |  |

## Operations

see section Graphs.
Implementation
see section Graphs.

## Parameterized Ugraphs (UGRAPH)

## Definition

A parameterized undirected graph $G$ is an undirected graph whose nodes and contain additional (user defined) data (cf. Parameterized Graphs). Every node contains an element of a data type vtype, called the node type of $G$ and every edge contains an element of a data type etype called the edge type of $G$.
\#include < LEDA/ugraph.h >

UGRAPH<vtype,etype> $U$ creates an instance $U$ of type ugraph and initializes it to the empty undirected graph.

## Operations

see section Parameterized Graphs.

## Implementation

see section Parameterized Graphs.

## Node Arrays ( node_array )

Definition

An instance $A$ of the parameterized data type node array<E> is a partial mapping from the node set of a graph $G$ to the set of variables of type $E$, called the element type of the array. The domain $/$ of $A$ is called the index set of $A$ and $A(v)$ is called the element at position $v$. $A$ is said to be valid for all nodes in $I$.
\#include < LEDA/node_array.h >

## Creation

| node_array<E> | A | creates an instance $A$ of type node_array<E> with empty index <br> set. |
| :--- | :--- | :--- |
| node_array<E> | A(graph |  |
| G) |  |  |$\quad$| creates an instance $A$ of type node_array<E> and initializes the |
| :--- |
| index set of $A$ to the current node set of graph $G$. |

## Operations

| E\& A[node v] | returns the variable $A(v)$. <br> Precondition $A$ must be valid for $v$. |
| :--- | :--- |
| void A.init(graph G) | sets the index set $I$ of $A$ to the node set of $G$, i.e., makes $A$ valid for <br> all nodes of $G$. |
| void A.init(graph G, E | makes $A$ valid for all nodes of $G$ and sets $A(v)=x$ for all nodes $v$ of <br> G. |

## Implementation

Node arrays for a graph $G$ are implemented by C++vectors and an internal numbering of the nodes and edges of $G$. The access operation takes constant time, init takes time $O(n)$, where $n$ is the number of nodes in $G$. The space requirement is $O(n)$.

Remark: A node array is only valid for a bounded number of nodes of $G$. This number is either the number of nodes of $G$ at the moment of creation of the array or it is explicitely set by the user. Dynamic node arrays can be realized by node maps (cf. section Node Maps).

## Edge Arrays ( edge_array )

## Definition

An instance $A$ of the parameterized data type edge_array<E> is a partial mapping from the edge set of a graph $G$ to the set of variables of type $E$, called the element type of the array. The domain I of $A$ is called the index set of $A$ and $A(e)$ is called the element at position $e . A$ is said to be valid for all edges in $I$.
\#include < LEDA/edge_array.h >

## Creation

| edge_array<E> | A | creates an instance $A$ of type edge_array<E> with empty index set. |
| :---: | :---: | :---: |
| edge_array<E> | A(graph G) | creates an instance $A$ of type edge_array<E> and initializes the index set of $A$ to be the current edge set of graph $G$. |
| edge_array<E> | $\begin{aligned} & \text { A(graph G, } \\ & \mathrm{E} \text { x) } \end{aligned}$ | creates an instance $A$ of type edge_array< $E>$, sets the index set of $A$ to the current edge set of graph $G$ and initializes $A(v)$ with $x$ for all edges $v$ of $G$. |
| edge_array<E> | A(graph G, int n, E x) | creates an instance $A$ of type edge_array<E> valid for up to $n$ edges of graph $G$ and initializes $A(\bar{e})$ with $x$ for all edges $e$ of G. <br> Precondition $n>=\|E\|$. <br> $A$ is also valid for the next $n-\|E\|$ edges added to $G$. |

## Operations

| E\& A[edge e] | returns the variable $A(e)$. <br> Precondition $A$ must be valid for $e$. |
| :--- | :--- |
| void A.init(graph G) | sets the index set $/$ of $A$ to the edge set of $G$, i.e., makes $A$ valid for <br> all edges of $G$. |
| void A.init(graph G, E | makes $A$ valid for all edges of $G$ and sets $A(e)=x$ for all edges $e$ of <br> X) |

## Implementation

Edge arrays for a graph $G$ are implemented by $\mathrm{C}++$ vectors and an internal numbering of the nodes and edges of $G$. The access operation takes constant time, init takes time $O(n)$, where $n$ is the number of edges in $G$. The space requirement is $O(n)$.

Remark: An edge array is only valid for a bounded number of edges of $G$. This number is either the number of edges of $G$ at the moment of creation of the array or it is explicitely set by the user. Dynamic edge arrays can be realized by edge maps (cf. section Edge Maps).

## Graph Windows ( GraphWin )

## Definition

GraphWin combines the two types graph and window and forms a bridge between the graph data types and algorithms and the graphics interface of LEDA. GraphWin can easily be used in LEDA programs for constructing, displaying and manipulating graphs and for animating and debugging graph algorithms.
\#include < LEDA/graphwin.h >

## Creation

GraphWin gw(graph\& G, const char* win_label="")
creates a graph window for graph $G$ with a display window of default size and frame label win_label.
GraphWin gw (window\& as above, but $W$ is used as display window. W)

## Operations

## a) Window Operations

| void <br> bool$\quad$gw.display() <br> gw.edit() | displays gw at default position. <br> enters the edit mode of GraphWin that allows to change the <br> graph interactively by operations associated with certain mouse <br> events or by choosing operations from the windows menu bar <br> (see section edit-mode for a description of the available <br> commands and operations). Edit mode is terminated by either <br> pressing the done button or by selecting exit from the file menu. <br> In the first case the result of the edit operation is true and in the <br> latter case the result is false. |
| :--- | :--- | :--- |
| as above, but displays the window at default position. |  |
| closes the window. |  |

b) Graph Operations
void gw.clear graph()
deletes all nodes and egdes.
graph\& gw.get_graph() returns a reference of the graph of $g w$.
void gw.update_graph() this operation has to be called after any update operation that has been performed directly (not by Graph Win) on the underlying graph, e.g., deleting or inserting nodes or edges.

## I) Miscellaneous

void gw.set_graph(graph\& makes $G$ the graph of $g w$.
G)
bool gw.wait() waits until the done button is pressed (true returned) or exit is selected from the file menu (false returned).
bool gw.wait(const char* displays msg and waits until the done button is pressed (true msg ) returned) or exit is selected from the file menu (false returned).
bool gw.wait(float sec, const char* msg="")
as above but waits no longer than sec seconds returns ?? if neither button was pressed within this time interval.
void gw.acknowledge(string displays string $s$ and asks for acknowledgement. s)
node gw.ask_node() asks the user to select a node with the left mouse button. If a node is selected it is returned otherwise nil is returned.
edge gw.ask_edge() asks the user to select an edge with the left mouse button. If an edge is selected it is returned otherwise nil is returned.

